Well-typed Music Does Not Sound Wrong

(Experience Report)

Dmitrij Szamozvancev
University of Cambridge
ds709@cam.ac.uk

Michael B. Gale
University of Warwick
m.gale@warwick.ac.uk

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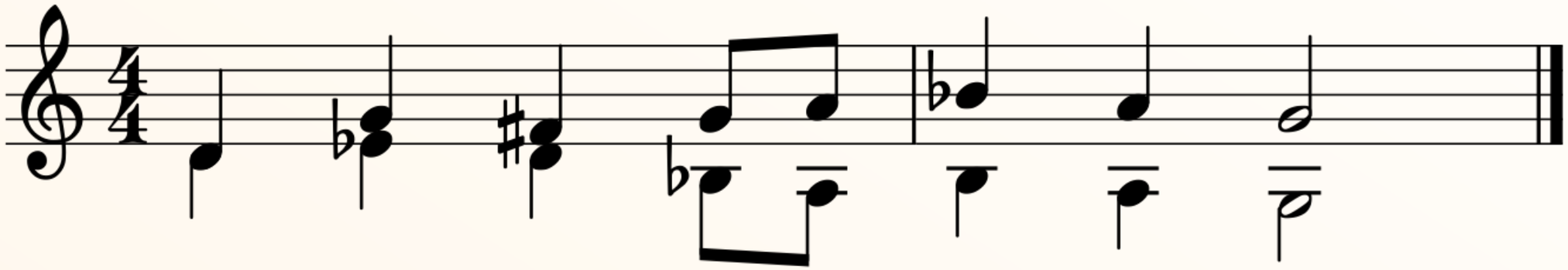
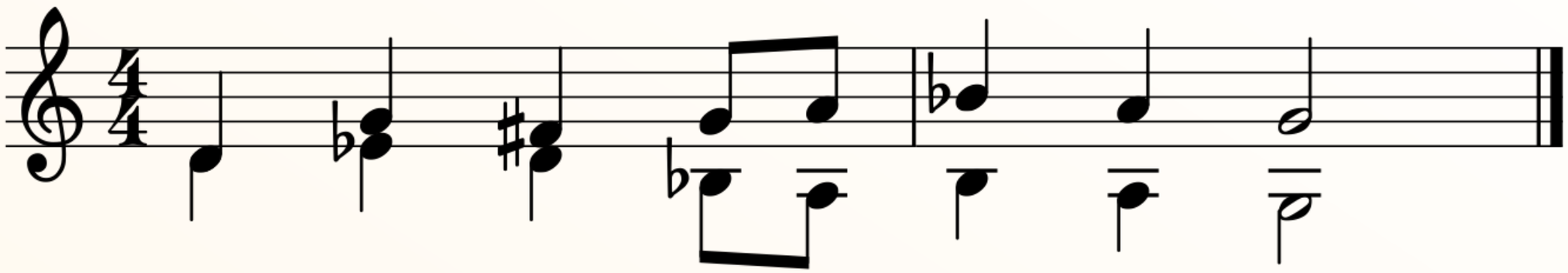
Does this piece sound good?



Mezzo example



```
import Mezzo

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  :|: a en :|: bf qn :|: a  qn :|: g hn

v2 = d  qn :|: ef qn :|: d  qn :|: bf_ en
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main = playLive (v1 :-: v2)
```

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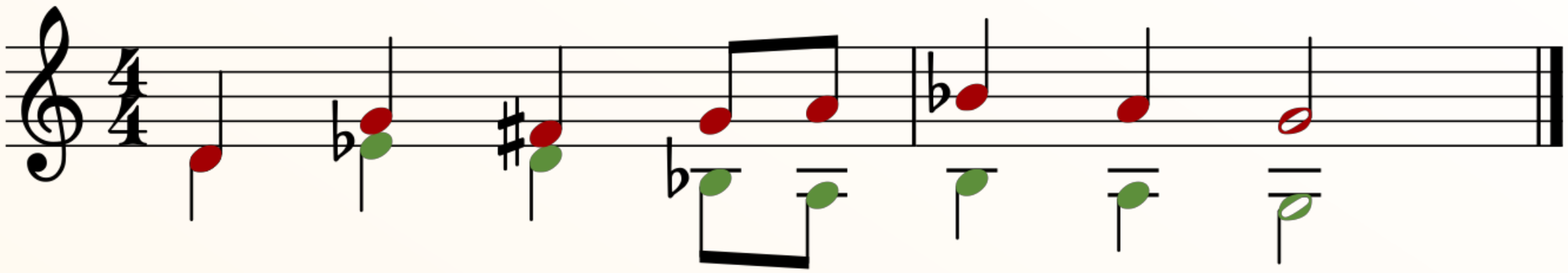

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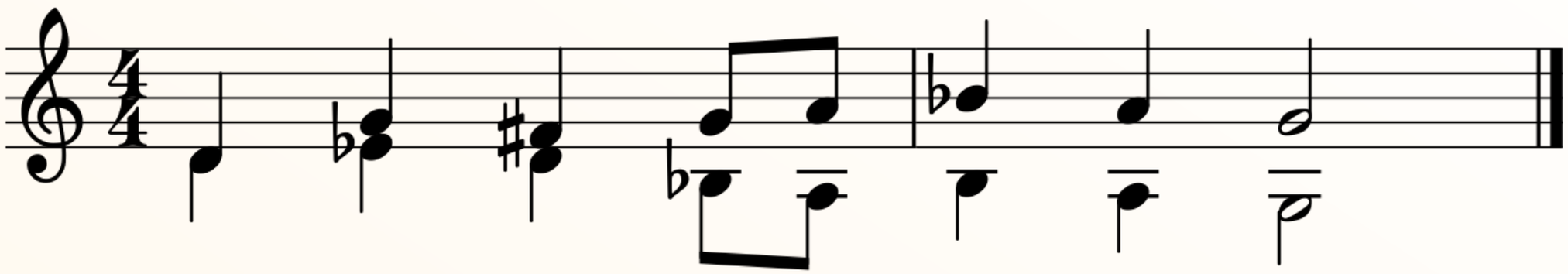

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- Major sevenths are not permitted in harmony:
  Bb and B_
- Direct motion into a perfect octave is forbidden:
  Bb and B_, then A and A_
- Parallel octaves are forbidden:
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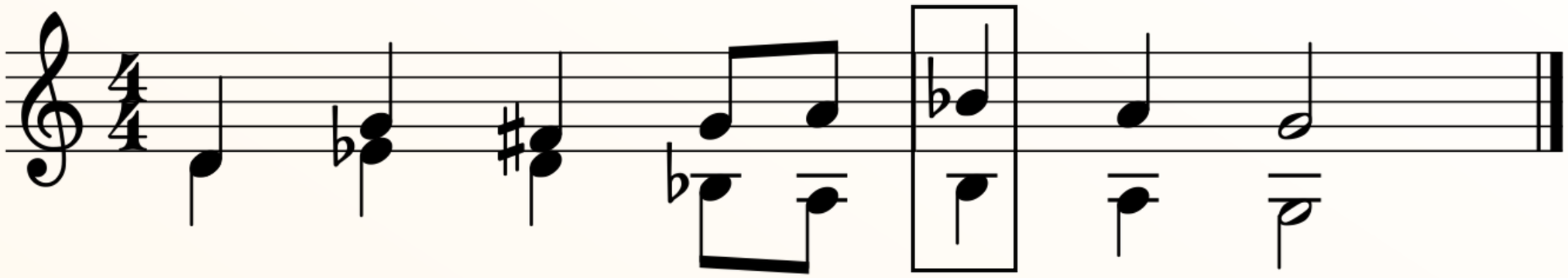

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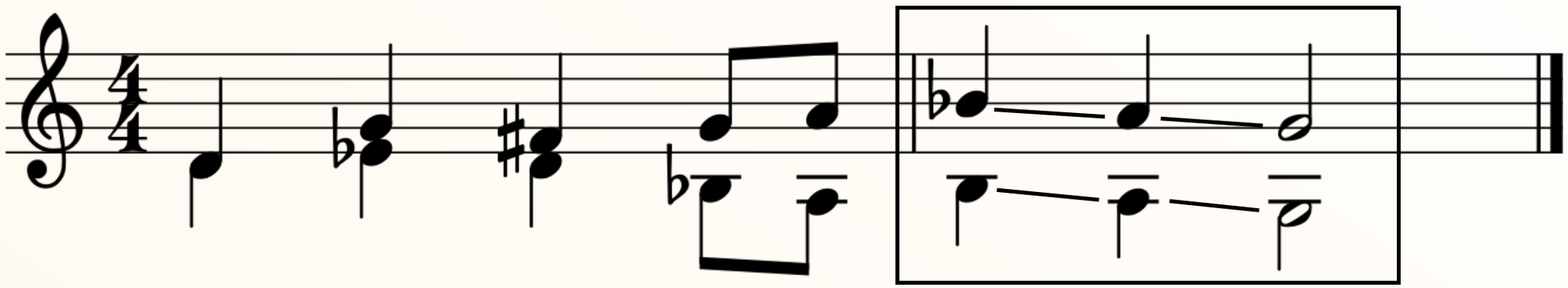

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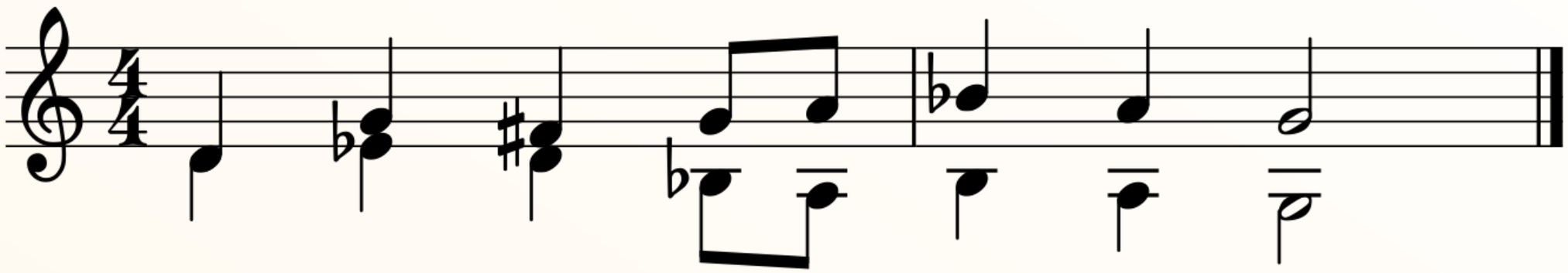
```
1
2   import Mezzo
3
4   comp = d qn :-: d qn :|: g qn :-: ef qn :|: fs qn :-: d qn
5      :|: g en :-: bf_ en :|: a en :-: a_ en :|: bf qn :-: b_ qn
6      :|: a qn :-: a_ qn :|: g hn :-: g_ hn
7
8   main = playLive' $ score setRuleSet strict withMusic comp
9
```

IDE-Haskell Linter

Error Warning Lint Build Test Repl Not set Auto

Mezzo example



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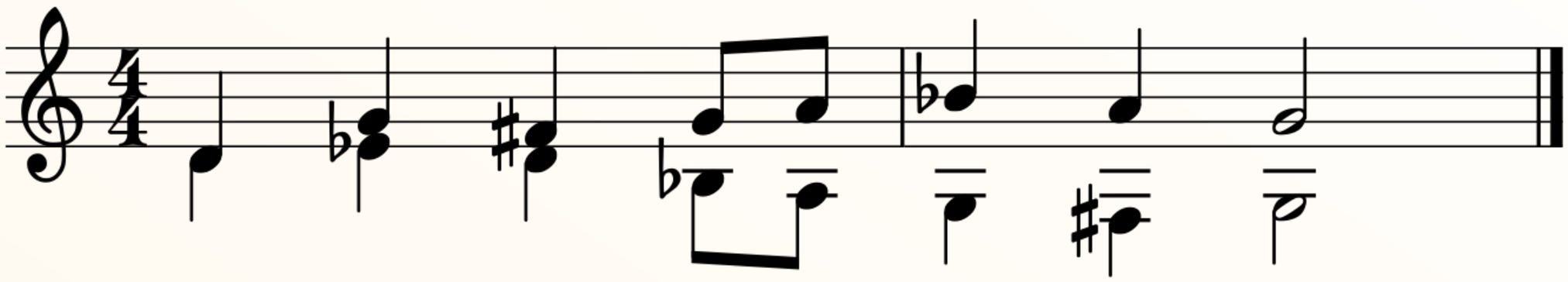

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main = playLive (v1 :-: v2)
```

Music theory

Western tonal music is governed by rules:

- What notes sound good together,
  or in sequence
- How voices should interact
- How a piece should be structured

Learning and following rules requires
care, attention, and time

Mezzo

A Haskell EDSL for music composition

Maintains a static model of music

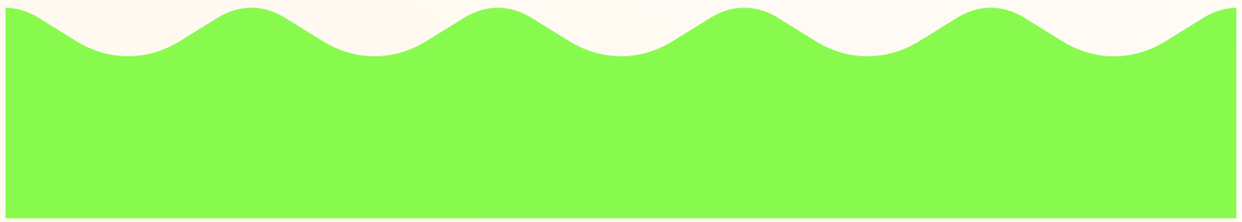
A dependently typed music algebra

Converts composition mistakes into type errors

Compiler errors describe the nature and
location of mistakes

Behind the scenes

The Mezzo recipe

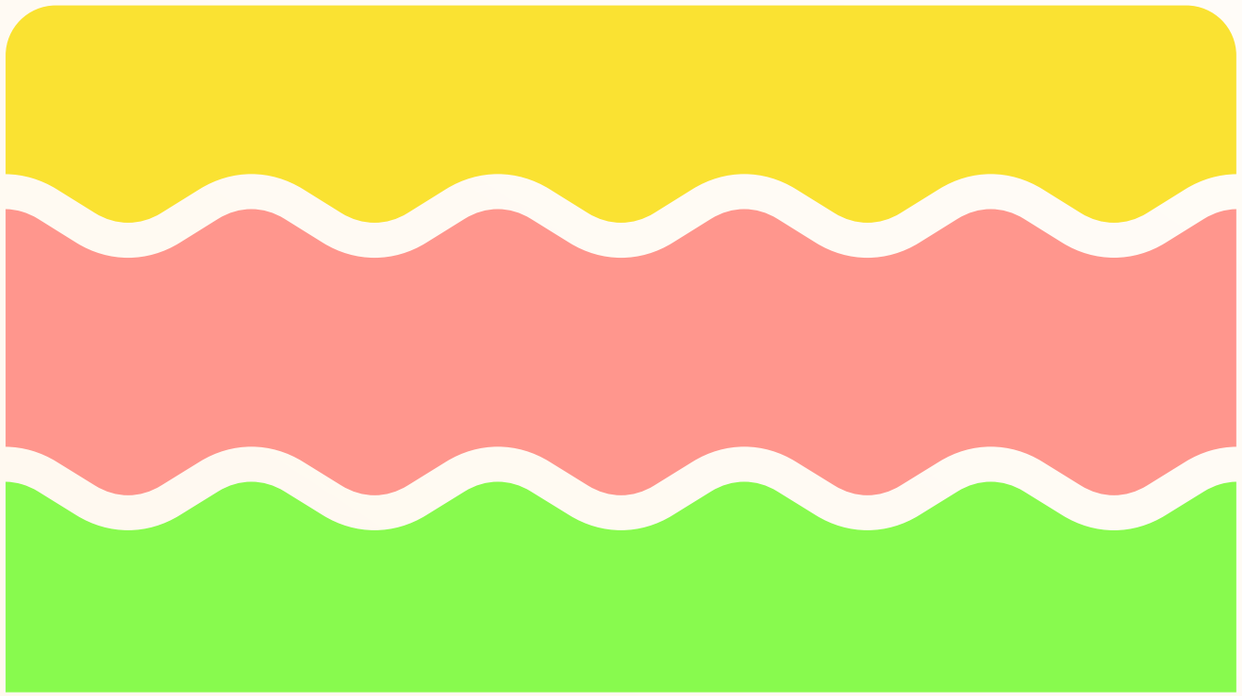


1. Take the Haskore music algebra

The Mezzo recipe

2. Add some dependent types

The Mezzo recipe



3. Hide everything under an EDSL

The Mezzo recipe

4. Add MIDI export functionality

The Haskore music algebra

An algebraic description of music

Primitives and two composition operators

$$M ::= \text{NOTE} \mid \text{REST} \mid M :-: M \mid M :|: M$$

Primitive values

Harmonic composition

Melodic composition

The Haskore music algebra

An algebraic description of music

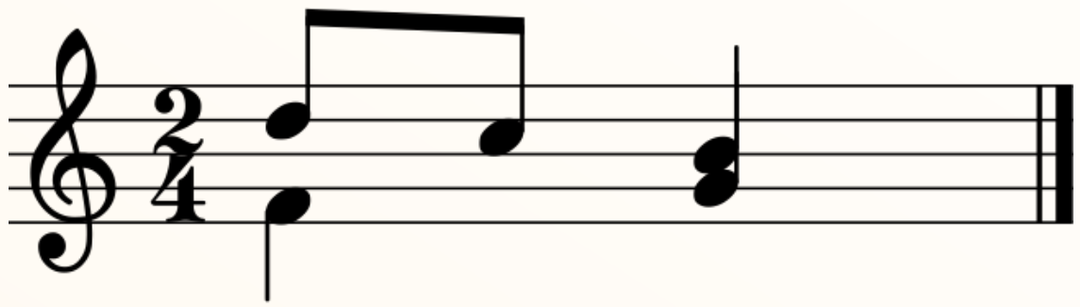
Primitives and two composition operators

$$M \ ::= \ \textsc{Note} \ | \ \textsc{Rest} \ | \ M :-: M \ | \ M :|: M$$

Primitive values | Harmonic composition | Melodic composition

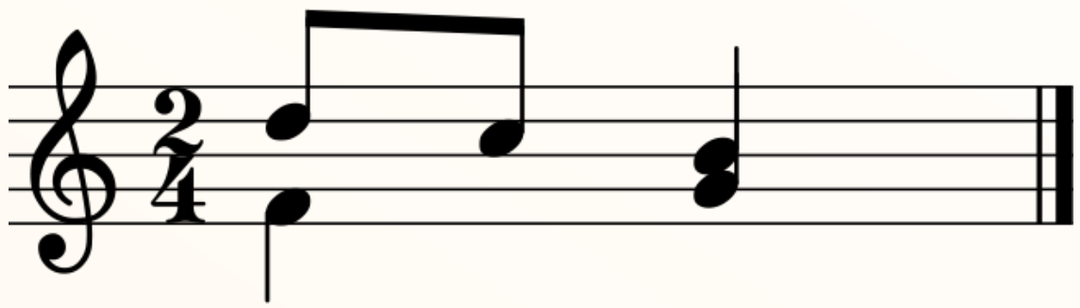
```
data Music = Note Pit Dur
           | Rest Dur
           | Music :-: Music
           | Music :|: Music
```

The Haskore music algebra



```
((D5 :|: C5) :-: F4) :|: (B4 :-: G4)
```

The Haskore music algebra



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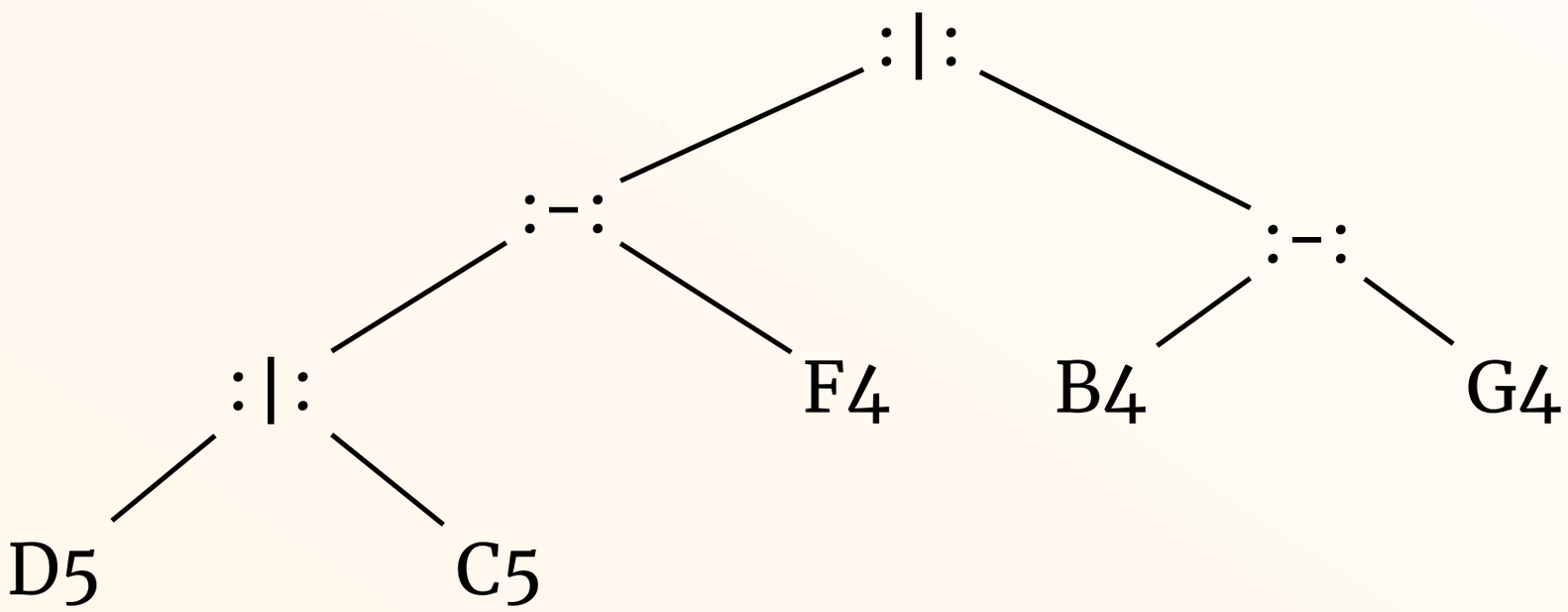
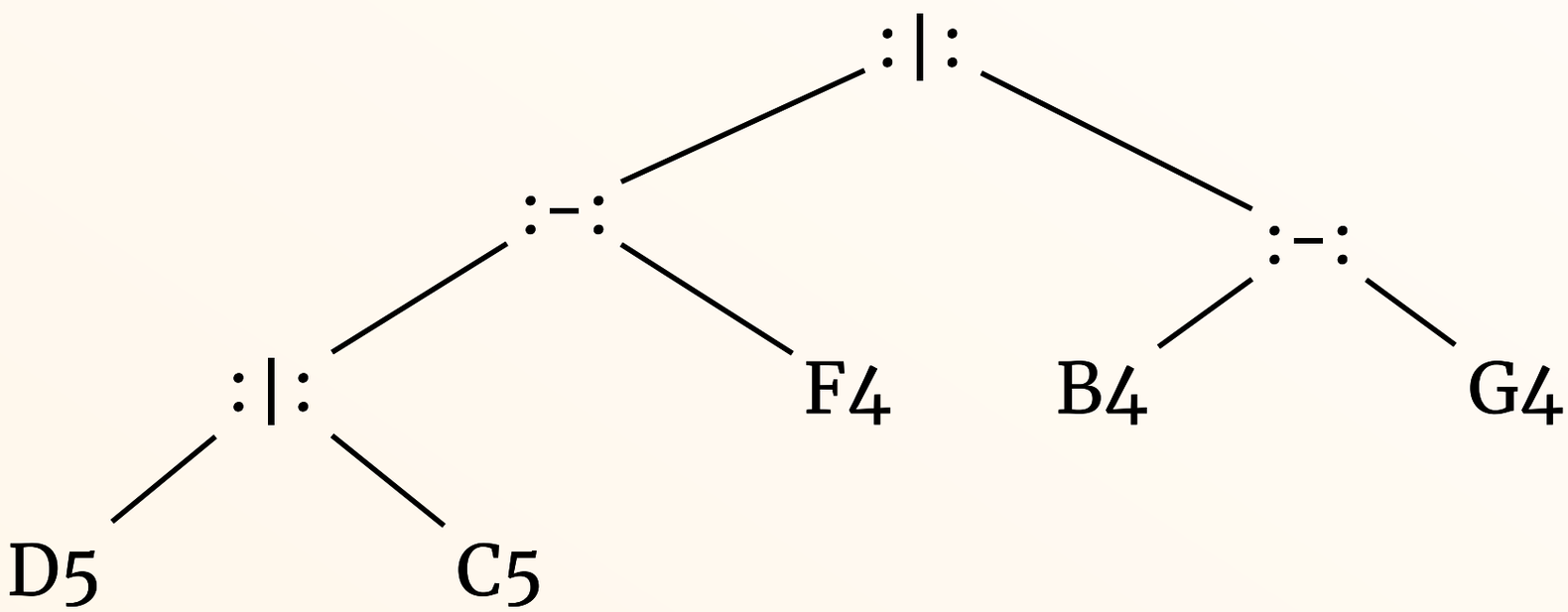


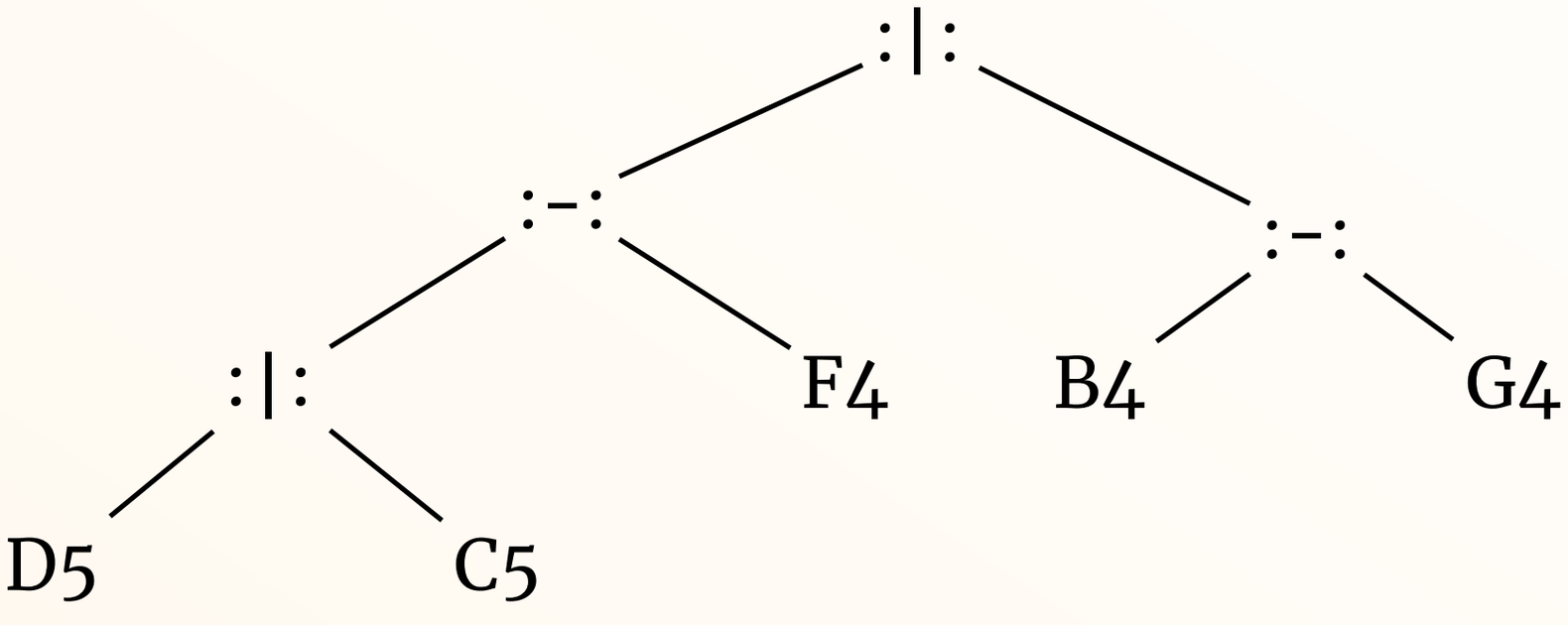
The Haskore music algebra

Type level

Term level



The Haskore music algebra



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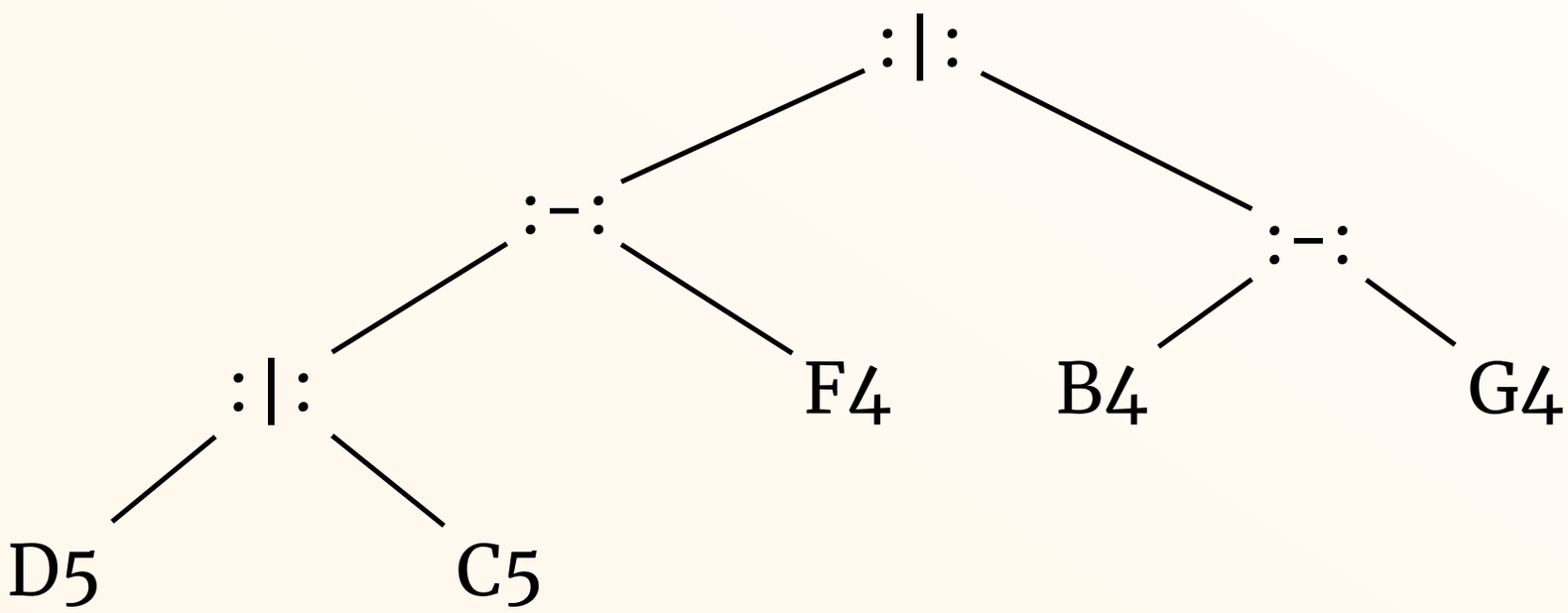

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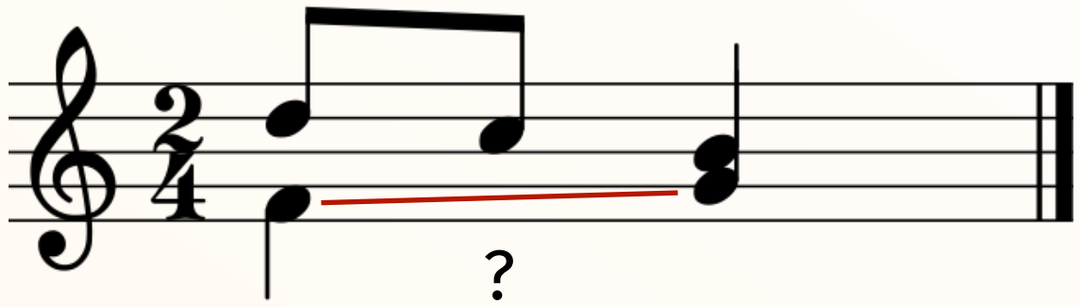
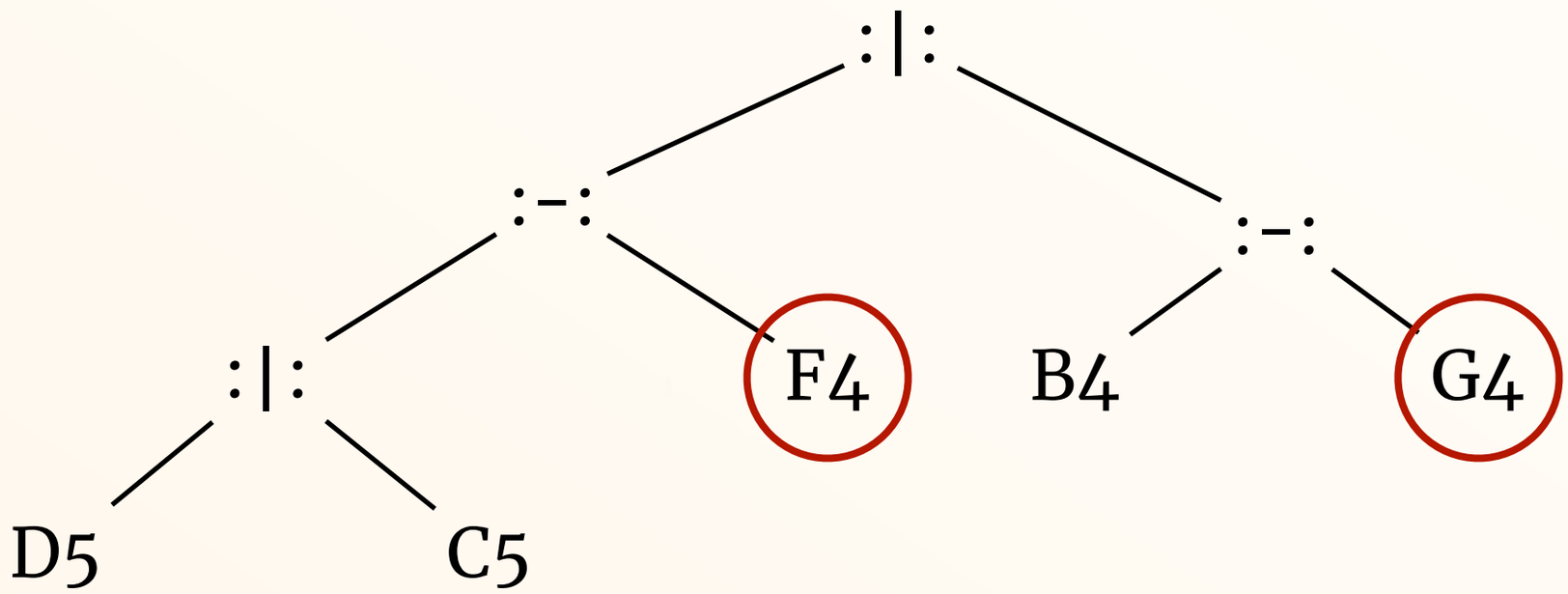
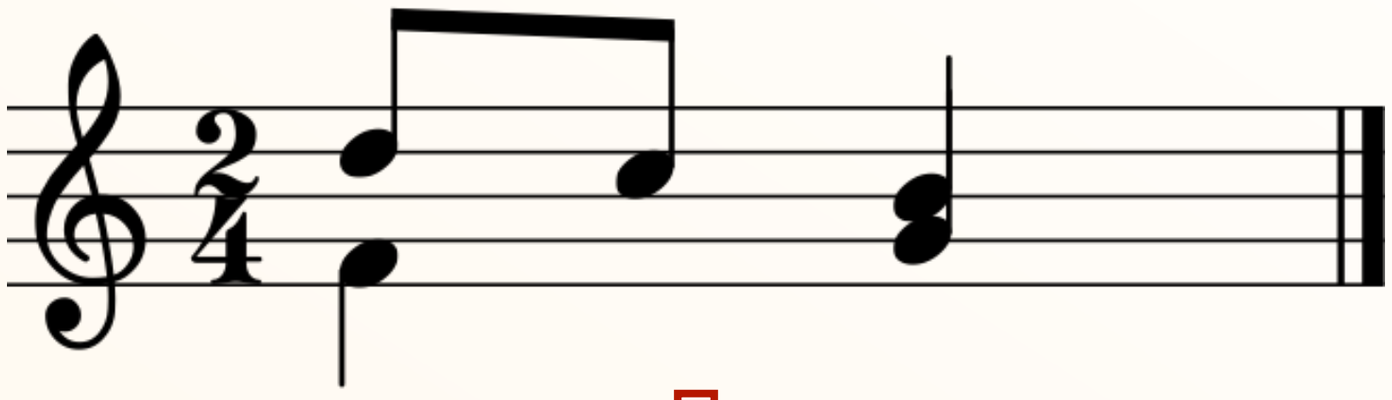




Intuitive for composition

Unsuitable for verification

Pitch matrix



♪ D5	♪ C5	♩ B4
♪ F4	♪ F4	♩ G4

The Haskore music algebra

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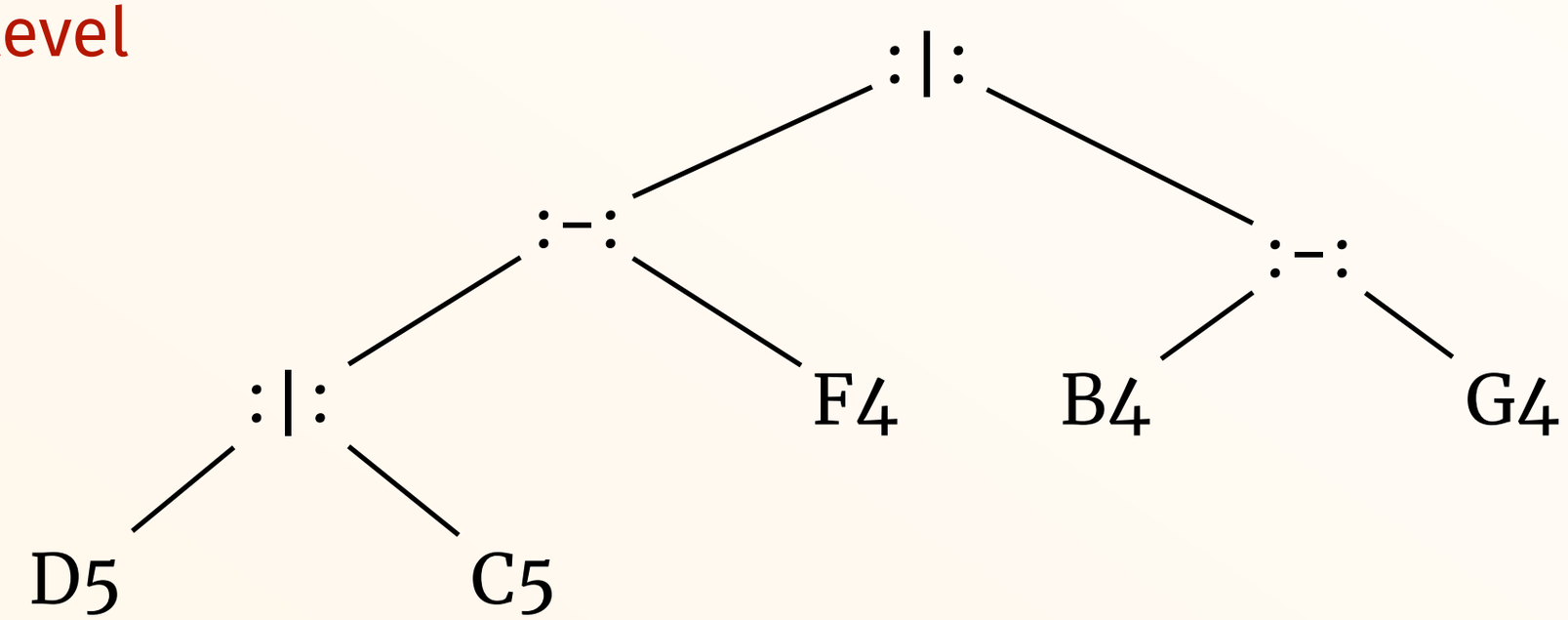


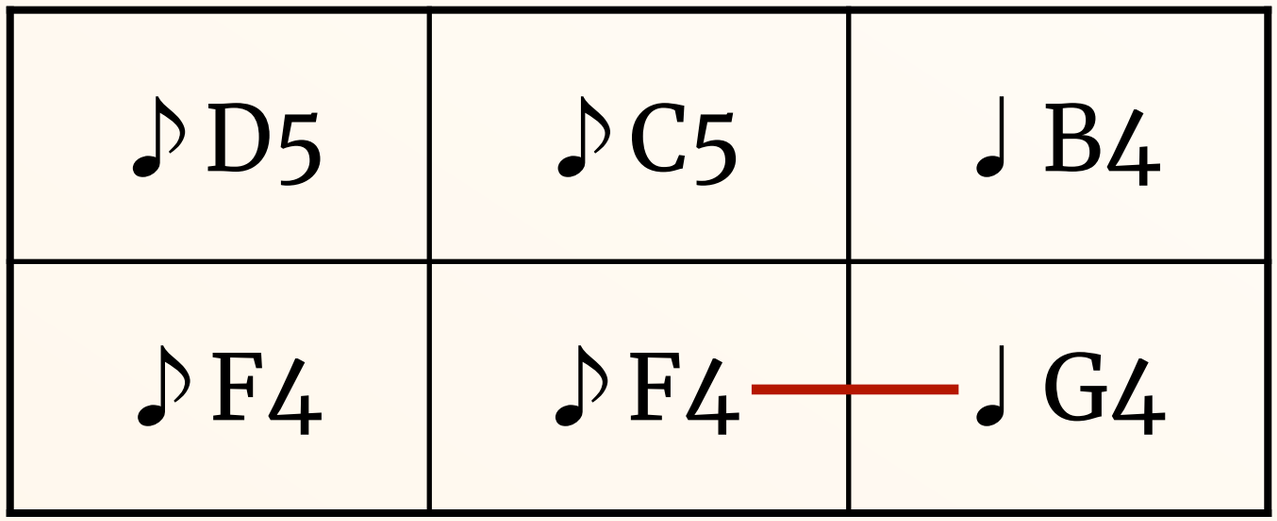
Pitch matrix

Alternative music format, suitable for verification

Has a clear, rigid, non-hierarchical structure

Reflects the visual layout of the score

Obvious relationship between parallel and successive notes



Pitch matrix

Our aim is to store the pitch matrix on the type level

Enables static verification of the rules

Need to enforce invariance of dimensions

A simple type-level list of lists would not suffice

Length-indexed vectors?

♪ D5	♪ C5	♩ B4
♪ F4	♪ F4	♩ G4

Where GHC 8 comes into play

Problem: keeping a size-indexed matrix on the type level requires GADT promotion

#7961 closed feature request (fixed)

Opened 4 years ago
Closed 21 months ago

Remove restrictions on promoting GADT's

Reported by:	danharaj	Owned by:	
Priority:	normal	Milestone:	~~8.0.1~~
Component:	Compiler	Version:	7.6.3
Keywords:		Cc:	eir@..., adam.gundry@..., jstolarek, william.knop.nospam@...
Operating System:	Unknown/Multiple	Architecture:	Unknown/Multiple
Type of failure:	None/Unknown	Test Case:	dependent/should_compile/TypeLevelVec
Blocked By:		Blocking:	
Related Tickets:	#6024	Differential Rev(s):	Phab:D808
Wiki Page:			

Where GHC 8 comes into play

Problem: keeping a size-indexed matrix on the type level requires GADT promotion

Changed 21 months ago by Richard Eisenberg <eir@…>

In 67465497/ghc:

```
Add kind equalities to GHC.

This implements the ideas originally put forward in
"System FC with Explicit Kind Equality" (ICFP'13).

There are several noteworthy changes with this patch:
 * We now have casts in types. These change the kind
   of a type. See new constructor `CastTy`.

 * All types and all constructors can be promoted.
   This includes GADT constructors. GADT pattern matches
   take place in type family equations. In Core,
   types can now be applied to coercions via the
   `CoercionTy` constructor.
```

Where GHC 8 comes into play

Problem: keeping a size-indexed matrix on the type level requires GADT promotion

Solution: upgrade to GHC 8!

With GHC 8, GADTs can be promoted just like any other type

Enabled by the `TypeInType` extension

Where GHC 8 comes into play

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Solution: upgrade to GHC 8!

With GHC 8, GADTs can be promoted just like any other type

Enabled by the `TypeInType` extension

```
data Vector :: Type → Nat → Type where
  None  :: Vector t 0
  (:--) :: t → Vector t (n - 1) → Vector t n
```

Dependent Haskore algebra

```
data Music where
  Note  :: Pit → Dur → Music
  Rest  :: Dur → Music
  (:-:) :: Music → Music → Music
  (:|:) :: Music → Music → Music
```

Dependent Haskore algebra

```
data Music :: ∀ n l. PitchMatrix n l → Type where
  Note  :: Pit p → Dur d → Music (FromPitch p d)
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  (:-:) :: Music m1 → Music m2 → Music (m1 +-+ m2)
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A vector of vectors of pitches

A promoted GADT

Dependent Haskore algebra

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```

Promoted musical values:

```
data PitchClass = C | D | E | F |…
data PitchType = Pitch PitchClass
                       Accidental Octave
```

Kind-constrained *proxies*:

```
data Pit (p :: PitchType) = Pit
```

Dependent Haskore algebra

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```

Type families for constructing
and combining pitch matrices

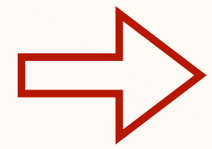
Concatenation respects the matrix dimensions

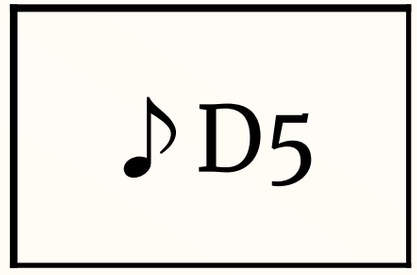
Made possible by the length-indexing in the kinds

```
type family (a :: PitchMatrix n k) +|+
            (b :: PitchMatrix n l)
            :: PitchMatrix n (k + l) where …
```

Dependent Haskore algebra

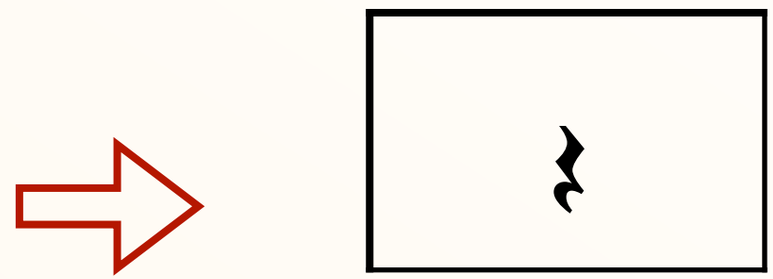
```
FromPitch D5 ♪
```





Dependent Haskore algebra

```
FromSilence ≀
```



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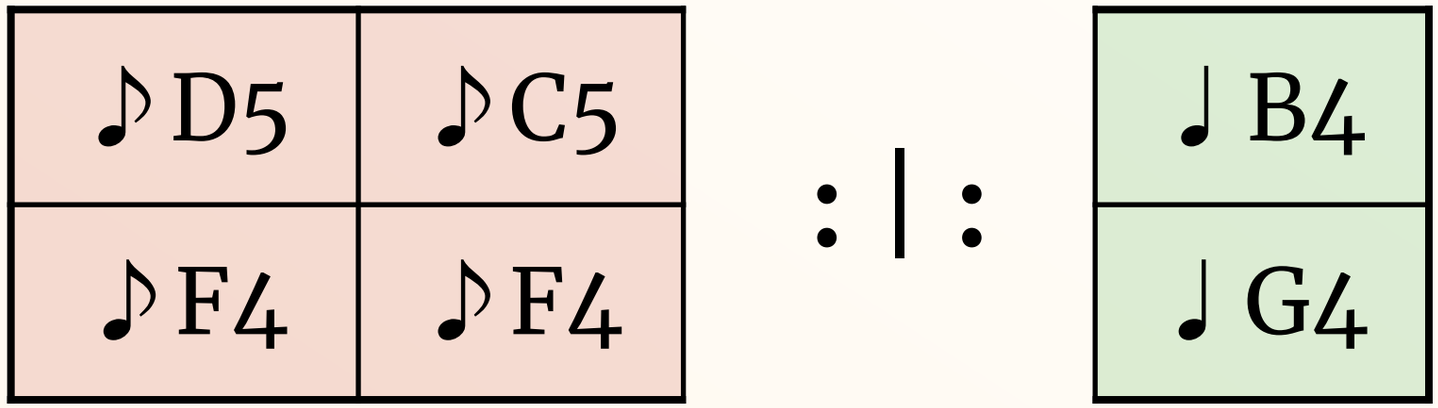
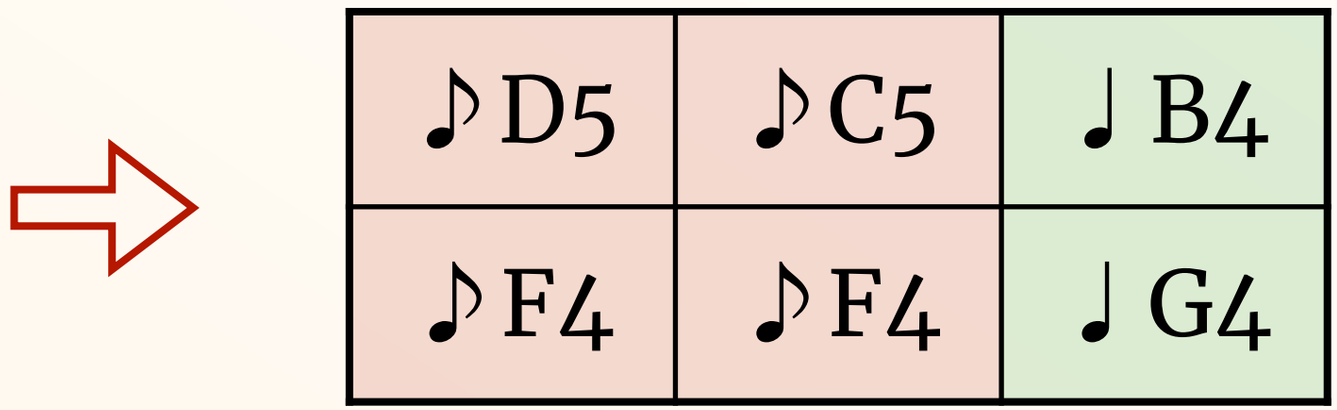


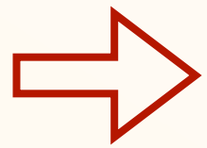


Dependent Haskore algebra

♪ D5	♪ C5	♩ B4

: – :

♩ F4	♩ G4



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```

We have static access to musical values through type variables

We impose *type class constraints* to limit the usage of the `Music` constructors

Musical constraints

```
data Music :: ∀ n l. PitchMatrix n l → Type where
  Note  :: ValidNote p d
        ⇒ Pit p → Dur d → Music (FromPitch p d)
  Rest  :: ValidRest d
        ⇒ Dur d → Music (FromSilence d)
  (:-:) :: ValidHarmComp m1 m2
        ⇒ Music m1 → Music m2 → Music (m1 +-+ m2)
(:|:) :: ValidMelComp  m1 m2
        ⇒ Music m1 → Music m2 → Music (m1 +|+ m2)
```

We have static access to musical values through type variables

We impose *type class constraints* to limit the usage of the `Music` constructors

Musical constraints

A series of inference rules as class hierarchies

“Axioms” specify valid and invalid intervals

Domain-specific error messages with
GHC’s custom compiler errors feature

```
class ValidMelInterval (i :: IntervalType)
instance TypeError
                (Text "Major sevenths forbidden.")
      ⇒ ValidMelInterval (Interval Maj Seventh)
instance {-# OVERLAPPABLE #-} ValidMelInterval i
```

Musical constraints

handle overlapping instances. In normal usage, *closed type classes* would not make much sense as the instances rarely overlap, but a separate construct acting as a closed *type predicate* could be useful for type-level programming and verification. Similarly, we often

GHC's custom compiler errors feature

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- New feature:
  Closed type classes

Musical constraints

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Haskellers leave no feature unabused

Musical constraints

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Musical constraints

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Rules propagate the interval axioms to the pitch matrix verification

```
class ValidMelLeap (p :: PitchType)
                   (q :: PitchType)
instance ValidMelInterval (MakeInterval p q)
      ⇒ ValidMelLeap p q
```

Musical constraints

Constraints connect the pitch matrix with the Haskore algebra

The rules are enforced any time a `Music` constructor is used

`ConstraintKinds` allows us to treat and manipulate constraints as types

Flexible means of validation, such as computed or partially applied constraints

The rule system is extensible and customisable

Constraints are further parameterised by *rule sets*

Rule sets

Allow for customisation of rule-checking

Not all genres of music follow the same rules

```
class RuleSet t where
  type MelConstraints  t m1 m2 :: Constraint
  type NoteConstraints t p  d  :: Constraint  ...

data Classical = Classical
instance RuleSet Classical where  ...

data Empty = Empty
instance RuleSet Empty where  ...
```

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class RuleSet t where
  type MelConstraints  t m1 m2 :: Constraint
  type NoteConstraints t p  d  :: Constraint  ...
```

Music values are parameterised by rule sets

Rule-checking behaviour can be modified dynamically

```
data Score = ∀ rs m. MkScore rs (Music rs m)

MkScore Classical (c qn :-: b qn) ✘
MkScore Empty     (c qn :-: b qn) ✓
```

Also in the paper

Details of the pitch matrix implementation

Treatment of duration and fragmentation

Construction of intervals

Some features of the EDSL

Note, chord and melody input

Reification and MIDI rendering

Summary and conclusions

Mezzo is a music composition library and EDSL with static rule-checking of musical scores

Exploits the term-type separation to manipulate two different models of music

No singletons required!

Built on the Haskore algebra, augmented with dependent types

Makes use of GADT promotion, type families and constraint kinds

Well-typed Music Does Not Sound Wrong

(Experience Report)

github.com/DimaSamoz/mezzo
hackage.haskell.org/package/mezzo

ds709@cam.ac.uk | m.gale@warwick.ac.uk

Advantages of static typing

Static, compile-time verification

Source location of mistakes

Two, distinct views of music

Haskore algebra for composition,
pitch matrix for verification

Simple term-level programming

Disadvantages of static typing

Complex type-level programming

But not much harder than doing
the same thing on the term level

Slower compilation

But time is saved on *finding* the mistakes

Term-type separation

Can be handled with standard Haskell techniques

Musical constraints

Musical constraints are implemented as a series of "inference rules" via type classes.

GHC's custom type error feature lets us specify which instances are invalid, and provide an explicit, domain-specific error message.

```
class ValidMelInterval (i :: IntervalType)
instance TypeError (Text "Major sevenths forbidden.")
      ⇒ ValidMelInterval (Interval Maj Seventh)
instance {-# OVERLAPPABLE #-} ValidMelInterval i
```

Class with no methods – an *open type predicate*. A type is either an instance (a valid melodic interval) or not (an invalid melodic interval).

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```

If `i` is unified with a major seventh interval, a type error is encountered (uses `GHC.TypeLits`).

Musical constraints

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instance TypeError (Text "Major sevenths forbidden.")
      ⇒ ValidMelInterval (Interval Maj Seventh)
instance {-# OVERLAPPABLE #-} ValidMelInterval i
```

Otherwise, the interval is valid. We need to handle overlapping instances, as Haskell type classes are open and not checked in order.

Musical constraints

```
class ValidMelInterval (i :: IntervalType)
instance TypeError (Text "Major sevenths forbidden.")
      ⇒ ValidMelInterval (Interval Maj Seventh)
instance {-# OVERLAPPABLE #-} ValidMelInterval i

class ValidMelLeap (p :: PitchType) (q :: PitchType)
instance ValidMelInterval (MakeInterval p q)
      ⇒ ValidMelLeap p q

class ValidMelAppend (v :: Voice l1) (w :: Voice l2)
instance ValidMelLeap (Last v) (Head w)
      ⇒ ValidMelAppend v w

class ValidMel (p :: PitchMatrix n k) (q :: PitchMatrix n l)
instance (ValidMelAppend v w, ValidMelConcat vs ws)
      ⇒ ValidMelConcat (v :-- vs) (w :-- ws)
```